DYNAMIC CHARACTERISTICS OF AN ELECTRIC dc ARC

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An equivalent electric circuit diagram is proposed for a dc plasmatron excited with alternating current. On the basis of this circuit diagram and with the aid of the dynamic characteristic, the phase angle of the excitation current is then calculated.

The dynamic characteristics of a dc arc have been studied experimentally in [1] by the method of small perturbations. An arc of approximately constant length was glowing in an argon atmosphere filling a channel between annular discs with a 6 mm inside diameter. The rate of gas flow was held constant at 0.03 g/sec. The direct current of the arc was varied over the 30-70 A range and was modulated with an alternating component, amplitude from 0 to 3.0 A and frequency from 0.2 to 20 kHz, by means of a special transistor device. The basic circuit diagram of the entire apparatus is shown in Fig. 1.

An interesting result was obtained from measurements of the phase shift between the alternating components of the dynamic volt–ampere characteristics (Fig. 2). At low frequencies (up to 2 kHz) the current lags the voltage, i.e., the phase shift is inductive. At a frequency above 2 kHz the current leads the voltage, i.e., the phase shift becomes capacitive. As the arc current increases, the magnitude of the phase shift at a given frequency decreases.

Little is found in the literature concerning phase shift calculations for an ac component superposed on a dc arc. In [2] such a calculation has been made for an arc with a horizontal volt–ampere characteristic (voltage independent of current). It is shown there* that in this case

$$\operatorname{tg} \varphi = k/\omega, \tag{1}$$

where φ denotes the phase shift angle between voltage and current, k is a parameter which depends on the properties of the fundamental discharge, and ω denotes the radian frequency of the alternating current. It follows from (1) that such an arc is inductive ($\varphi > 0$, i.e., the voltage leads the current).

The peculiar features of a discharge studied in [1] are, first of all, its U-shaped volt–ampere characteristic and, secondly, the arc glow in a channel with electrically conducting walls, which produces additional reactive components. Indeed, if the channel represents a smooth metallic cylinder, as shown in Fig. 1, then the inside surface of the channel and some discharge boundary may be viewed as the plates of a capacitor. The equivalent circuit diagram of a discharge corresponding to Fig. 1 may, therefore, be replaced by the diagram in Fig. 3.

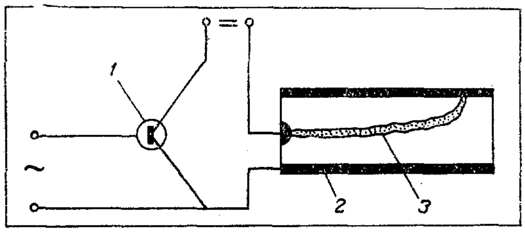

Fig. 1. Simplified schematic diagram of the test apparatus: (1) transistor, (2) channel, (3) arc.

If the channel walls represent an interelectrode insert made up of annular disks, then, as before, the inside surfaces of those disks and the discharge boundary

* Eq. (81.5) in [2], which corresponds to our Eq. (1) here, has the wrong sign.

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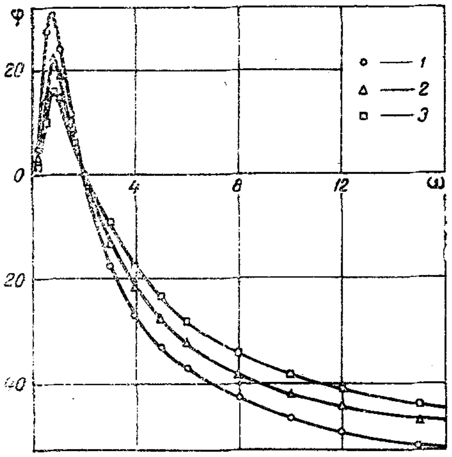


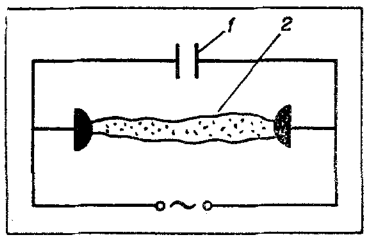


Fig. 2 Fig. 3

Fig. 2. Phase shift between voltage and current of the ac arc component: I = 30 A and U = 51 V (1), I = 50 A and U = 48 V (2), I = 70 A and U = 50 V (3).

Fig. 3. Equivalent electric circuit of an arc in a channel with electrically conducting walls: (1) equivalent capacitance, (2) arc.

will act as the plates of a capacitor, but an additional capacitance will also appear between the lateral surfaces of adjacent disks (Fig. 4). We will proceed on the basis of the simplified circuit diagram.

In order to calculate the equivalent capacitance, one must know the radial dimension of the discharge distribution zone at the electrode wall: this dimension is, evidently, of the order of the Debye radius. Inasmuch as the major potential drop between an arc and the channel wall occurs in just this zone, the transverse dimension of this zone can be estimated also from the results breakdown-voltage measurements in a plasmatron [3, 4]. According to those measurements, the dimension of the discharge distribution zone in a plasmatron with an arc glow in air is of the order of 1.0-0.1 mm. Consequently, a plasmatron 10 cm long and with a 1 cm inside diameter has an appreciable capacitance of the order of 25-250 nF.

In order to qualitatively describe the dynamic characteristic of an electric arc in a channel, we will use the approximate equations of an ac arc given in [5]:

$$\Omega \frac{d\Phi(\bar{h}_m)}{d\tau} = \beta^2 \frac{\int_1^{\bar{h}_m} \bar{\sigma}\, d\bar{h}}{2(\bar{h}_m - 1)} - 2(\bar{h}_m - 1), \tag{2}$$

$$\alpha = \beta \frac{\int_1^{\bar{h}_m} \bar{\sigma}\, d\bar{h}}{2(\bar{h}_m - 1)}, \tag{3}$$

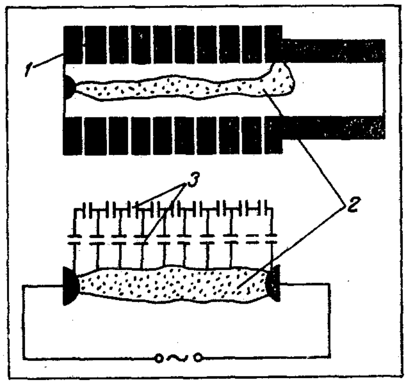
where

$$\bar{h} = h/h_1; \quad \bar{\rho} = \rho/\rho_1; \quad \bar{\sigma} = \sigma/\sigma_0;$$

$$\tau = \omega t; \quad \Omega = \frac{\omega \rho_1 c_{p_1} R^2}{\lambda_1};$$

$$\beta = ER\sqrt{\frac{\sigma_0 c_{p_1}}{\lambda_1 h_1}}; \quad \alpha = \frac{I}{2\pi R}\sqrt{\frac{c_{p_1}}{\lambda_1 h_1 \sigma_0}};$$

$$\Phi(\bar{h}_m) = \frac{\int_1^{\bar{h}_m} \left(\int_1^{\bar{h}} \bar{\rho}\, d\bar{h}\right) d\bar{h}.}{2(\bar{h}_m - 1)}.$$



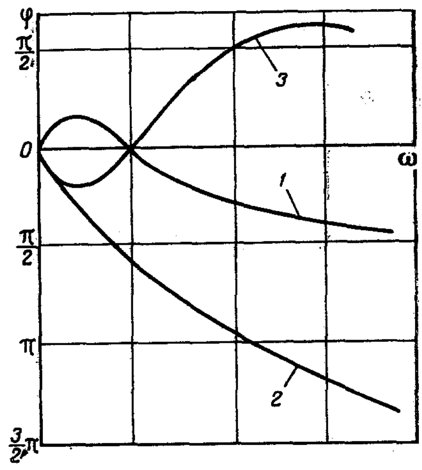


Fig. 4 Fig. 5

Fig. 4. Plasmatron with an interelectrode insert and its equivalent electric circuit: (1) annular disk, (2) arc, (3) equivalent capacitances.

Fig. 5. Frequency characteristic of the phase shift angle, as related to the arc properties according to (11).

Subscript 1 refers to the channel walls, subscript m refers to the channel axis, and σ_0 denotes the characteristic electrical conductivity of the medium.

According to [5], this system of equations plus the circuit equation describe all features of an ac arc which have been revealed in experiments: ignition and extinction peaks, current gaps, etc. In order to derive the equations describing the dynamic characteristics of a dc arc, we will determine the behavior of small excursions from the steady state which satisfies the condition $d\Phi/d\tau = 0$ (Φ = g, j, e). From (2) and (3) we easily obtain

$$\Omega a \frac{dg}{d\tau} = 2\alpha_0 e + (\beta_0^2 b - 2)\, g, \tag{4}$$

$$j\beta_0 = \alpha_0 e + \beta_0^2 b g, \tag{5}$$

$$a = \left(\frac{d\Phi}{d\bar{h}_m}\right)_0; \quad b = \left(\frac{d}{d\bar{h}_m}\frac{\int_1^{\bar{h}_m} \bar{\sigma}\, d\bar{h}}{2(\bar{h}_m - 1)}\right)_0.$$

Subscript 0 refers to the steady state and g, j, e are small excursions of $\bar{h}_m$, α, and β respectively. Eliminating g, we obtain an equation for the dynamic volt–ampere characteristic of a dc arc:

$$\Omega a \beta_0 \frac{dj}{d\tau} - \beta_0 (\beta_0^2 b - 2)\, j = \Omega a \alpha_0 \frac{de}{d\tau} + (\beta_0^2 b + 2)\, \alpha_0 e, \tag{6}$$

which differs from the one analyzed in [2] by the presence of a second term on the left-hand side. It can be shown that this term accounts for the electric field intensity in the arc being a function of the current. Indeed, the derivative of this volt–ampere characteristic ($d\Phi/d\tau = 0$) yields

$$\beta_0^2 b - 2 = -\frac{4x}{x+1}; \quad x = \frac{d \ln \beta}{d \ln \alpha} = \frac{d \ln E}{d \ln I}. \tag{7}$$

Thus, the sign of that additional term in Eq. (6) depends on the trend of the volt–ampere characteristic. If the latter is horizontal ($x = 0$), then (6) corresponds to the equation derived in [2].

The voltage drop across the capacitive circuit component will be written (in the notation introduced here) as

$$\beta = \frac{1}{z}\int_0^{\tau} \alpha' d\tau,$$

where α' is the component of α referred to the given circuit, $z = \omega l C/2\pi R^2\sigma_0$, l is the channel length, and C is the capacitance. For small excursions this equation becomes (z is assumed constant):

$$j' = z\frac{de}{d\tau}. \tag{8}$$

A phase shift is measured between the voltage (or quantity e) and the total current, which in terms defined here is

$$j_0 = j + j'. \tag{9}$$

Therefore, the equation for the dynamic volt–ampere characteristic of the circuit under consideration here becomes

$$\Omega a\beta_0 \frac{dj_0}{d\tau} - \beta_0\left(\beta_0^2 b - 2\right) j_0 = \Omega a\beta_0 z \frac{d^2 e}{d\tau^2}$$

$$+ \left[\Omega a\alpha_0 - \beta_0 z\left(\beta_0^2 b - 2\right)\right]\frac{de}{d\tau} + \alpha_0\left(\beta_0^2 b + 2\right)e. \tag{10}$$

Letting $j_0 = j_{00}e^{i\tau}$ and $e = e_0 e^{i(\tau + \varphi)}$, we find the phase angle between voltage and current:

$$\operatorname{tg}\varphi = \frac{2\Omega a\beta_0^2 b - z\dfrac{\beta_0}{\alpha_0}\left[(\Omega a)^2 + \left(\beta_0^2 b - 2\right)^2\right]}{(\Omega a)^2 - \left[\left(\beta_0^2 b\right)^2 - 4\right]}. \tag{11}$$

We will now examine the trend of φ as a function of the ac excitation frequency and as a function of the arc properties. On the rising portion of the volt–ampere characteristic, where $\beta_0^2 b < 2$ according to (7), the trend is indicated by curve l in Fig. 5. Such a relation corresponds to the experimental results (Fig. 2).

On the dropping portion of the volt–ampere characteristic, where $\beta_0^2 b > 2$, there are two possibilities. At a sufficiently small z (the denominator vanishes before the numerator) the phase shift angle is negative and approaches $(-(3\pi/2))$ as $\omega \to \infty$ (Fig. 5, curve 2).

With a sufficiently large z (the numerator vanishes before the denominator) the phase shift angle goes first negative at low frequencies, passing through a mimimum, and then increases through zero to $(+\pi/2)$ (Fig. 5, curve 3), passing then through a maximum.

We do not consider here the case of a large capacitance, where the numerator in (11) remains always positive.

It is to be noted that our circuit does not contain various additional resistances, as the resistance of lead wires and of the near-electrode arc regions, whose characteristics are different from those of the arc column. With these circuit components taken into consideration, the mode represented by curve 1 in Fig. 5 would extend somewhere into a drooping region of the volt–ampere characteristic. This is what probably happened in the experiments in [2], where the near-electrode voltage fall amounted to 50-100 V. As a result, the phase shift at a current of 30 A with the operating point of the arc on the drooping portion of the volt–ampere characteristic was of the same kind as on the rising portion of that characteristic (50 and 70 A).

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